Institute of Applied Problems of Physics

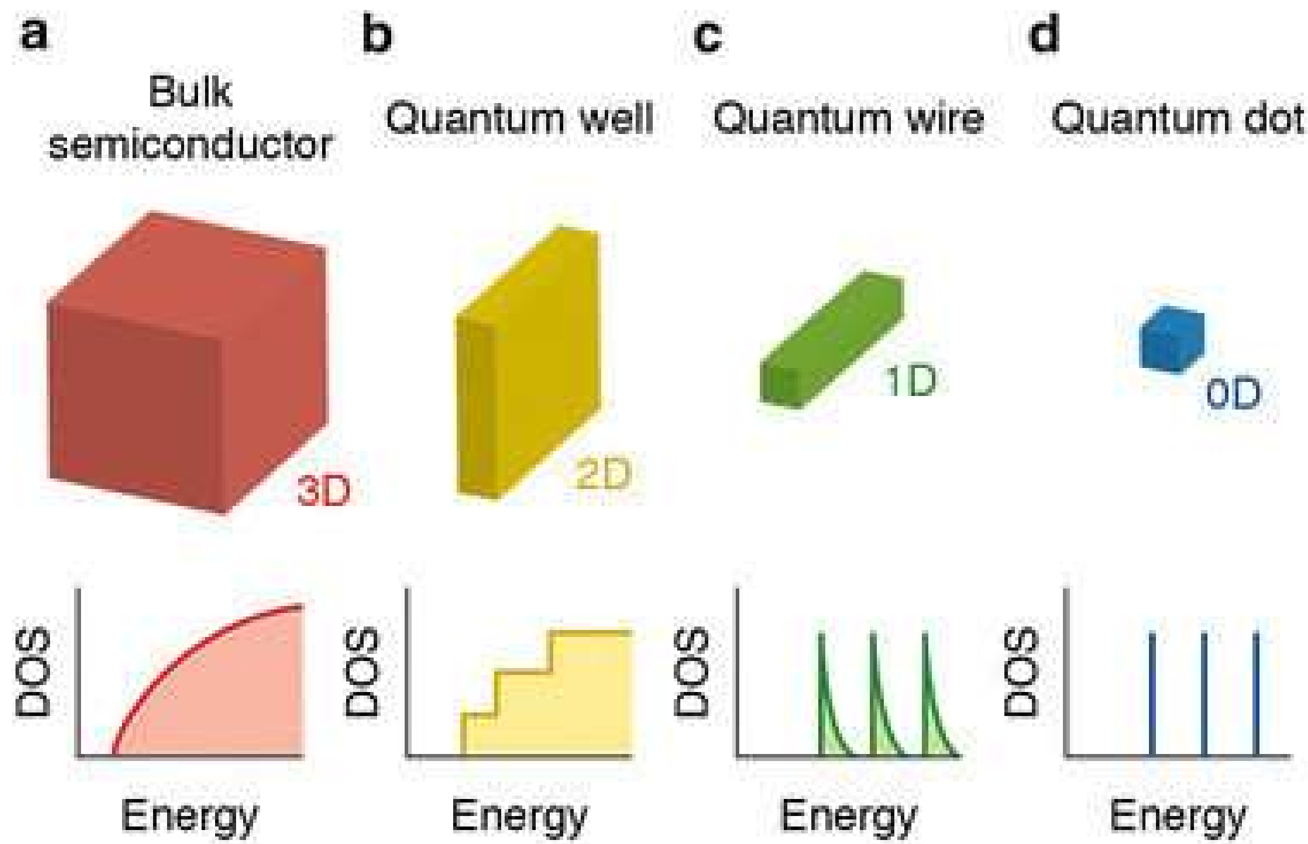
Laboratory of Physics of Nano- and Mesosystems

Implementation of exactly solvable multiparticle models for describing quantum dots with complex geometries

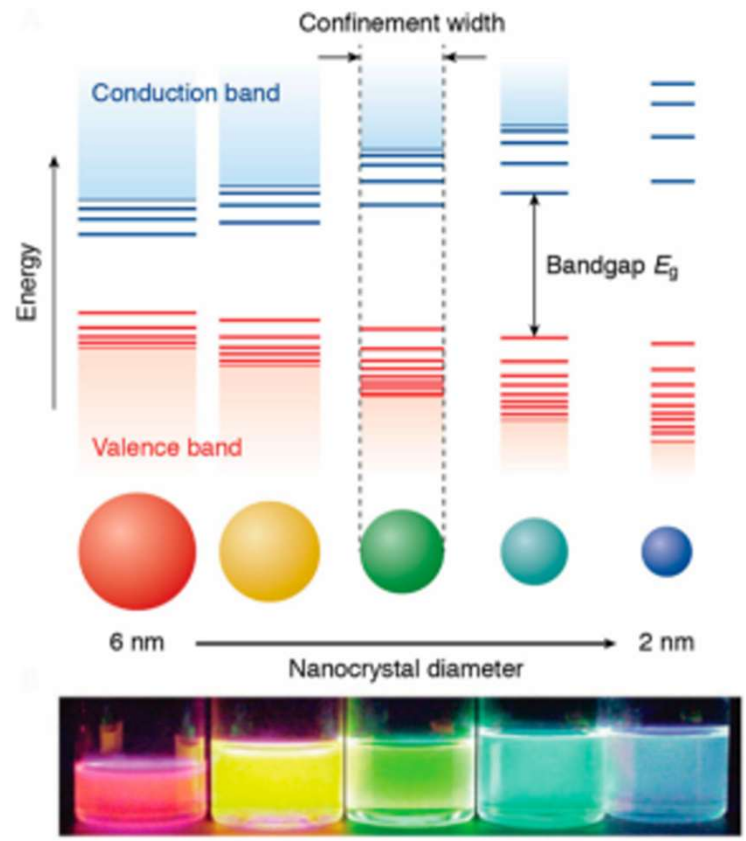
H.A. Sarkisyan

Maykop 2025

Types of nanostructures



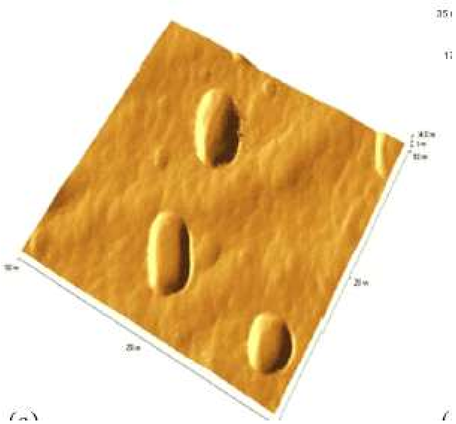
Quantum dot sizes: big, middle and small

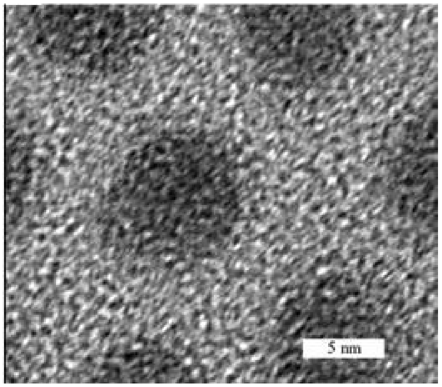


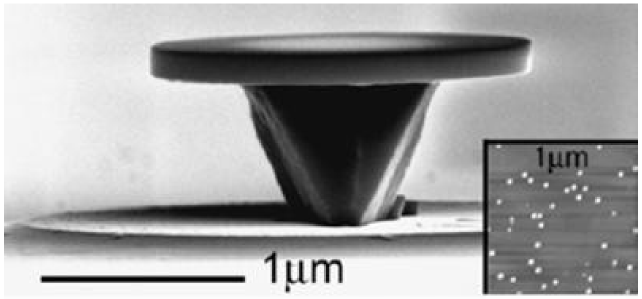
Bulk of Semiconductor (CdSe single crystal)

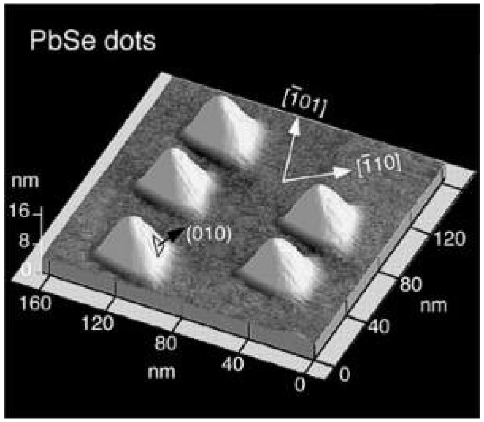
Nanocrystal of Semiconductor (CdSe Quantum Dots)

Quantum dot geometries

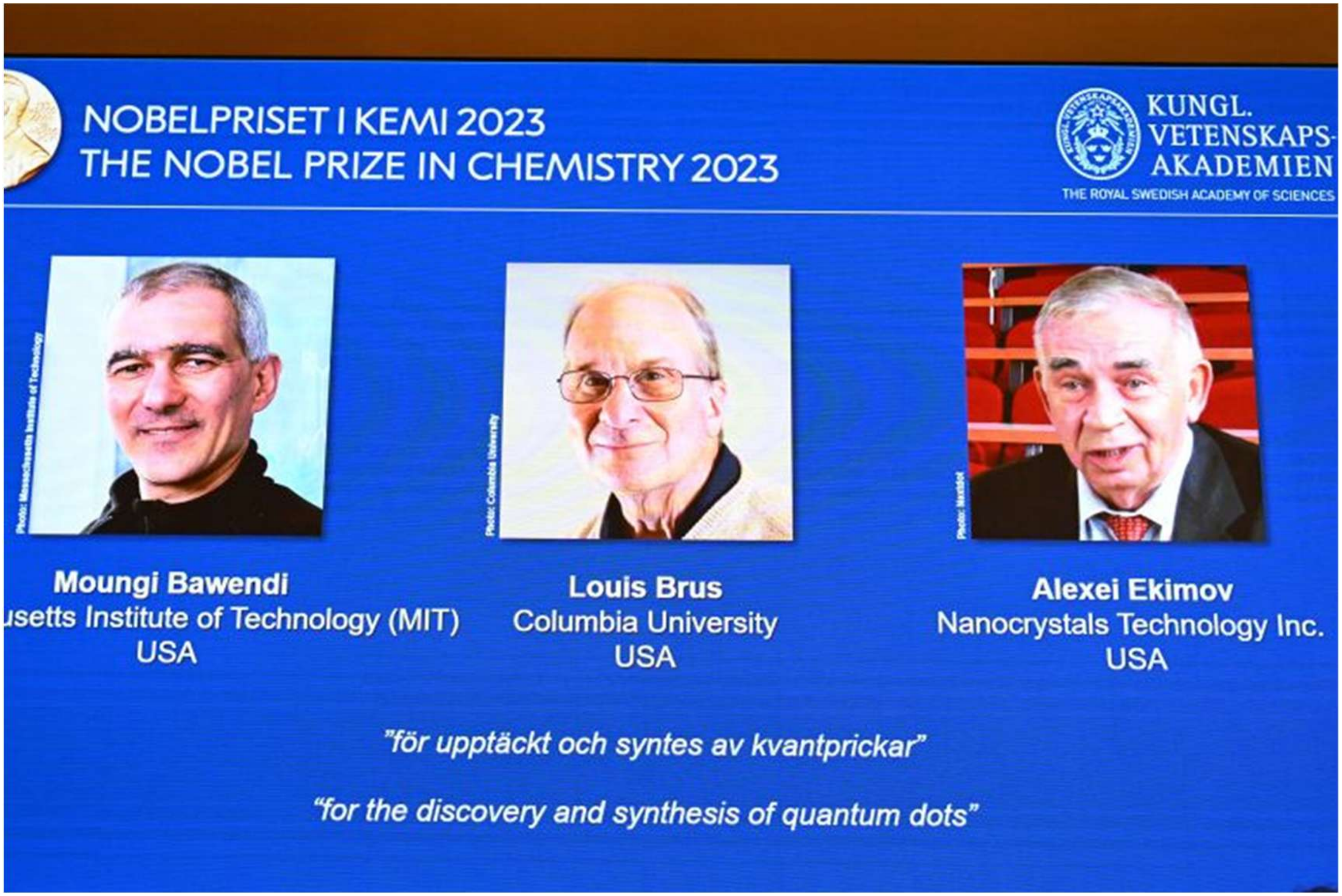








Nobel Prize in Chemistry 2023



Pioneering theoretical paper

Alexander Efros

Alexei Efros

Parabolic confining potential

Physica E 4 (1999) 231–237

The influence of inter-diffusion on electron states in quantum dots

J.A. Barker *, E.P. O'Reilly

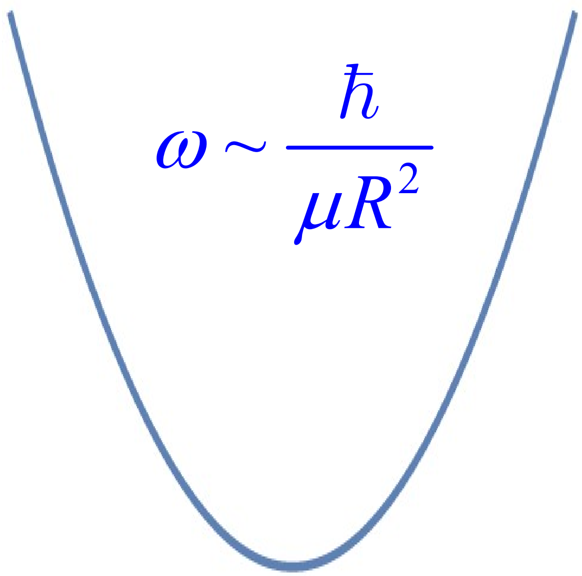
School of Physical Sciences, University of Surrey, Guildford, GU2 5XH, UK

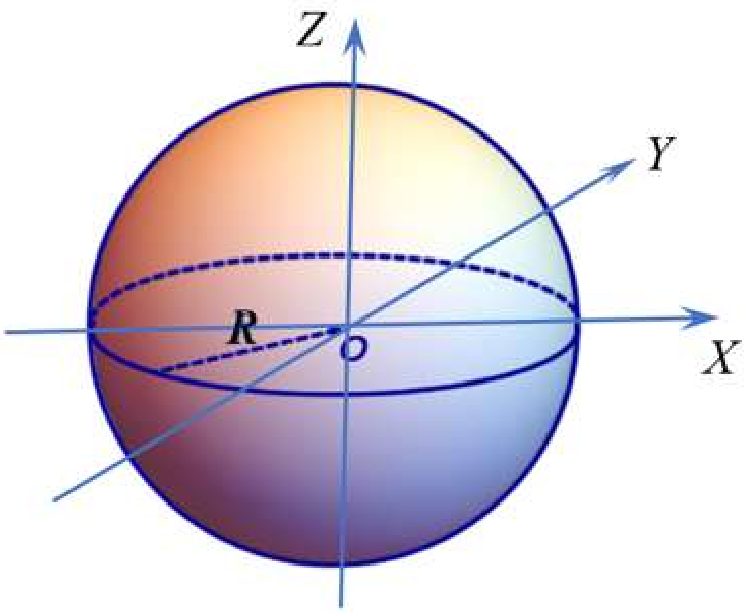
Received 8 July 1998; received in revised form 30 November 1998; accepted 15 December 1998

Parabolic confining potential

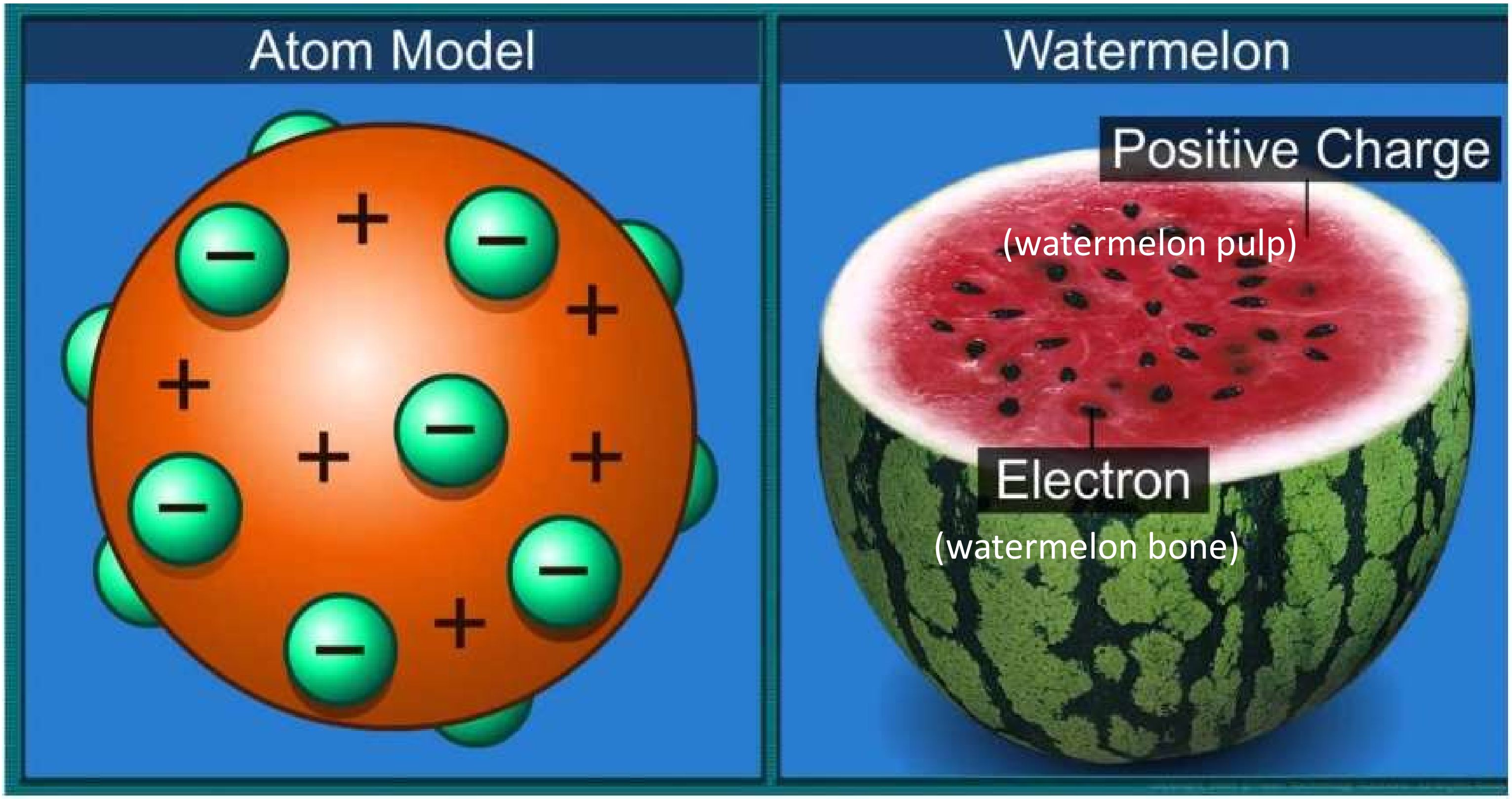
$$V_{conf}\left(\vec{r}\right)=\frac{\mu\omega^2 r^2}{2}$$

$$\omega\sim\frac{\hbar}{\mu R^2}$$





Thomson's atomic model



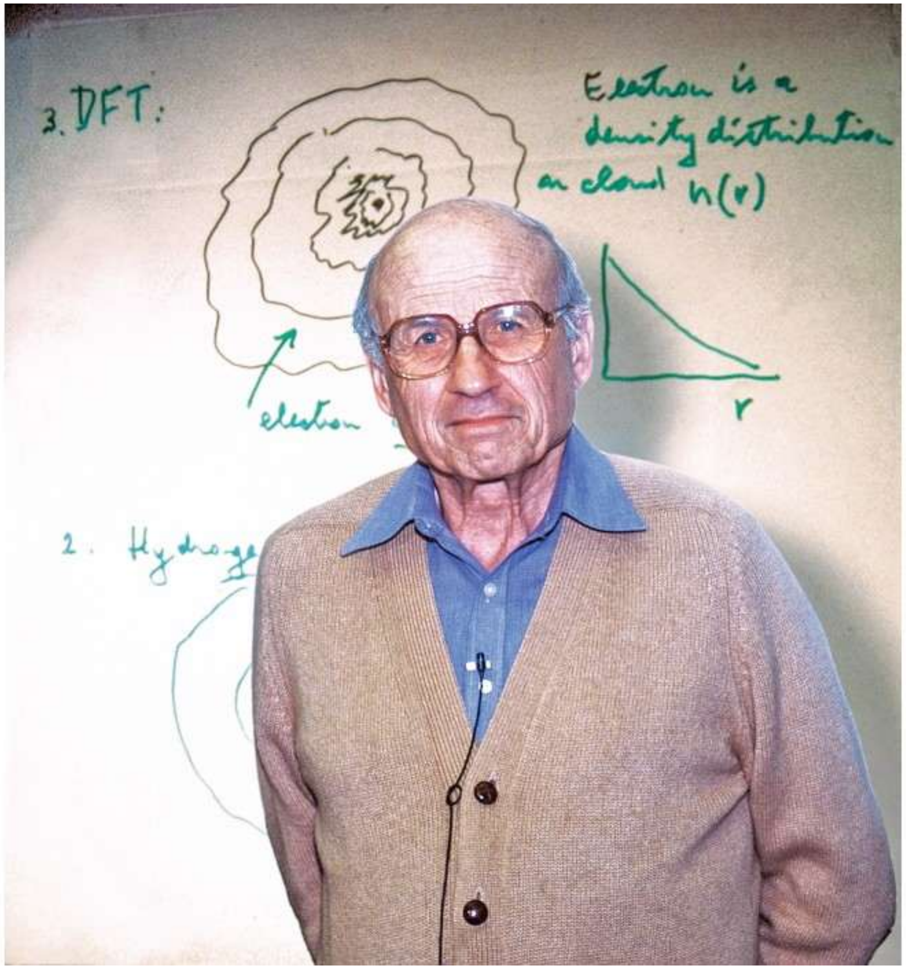
Formation of confining potential

$$\hat{V}_{conf}(r) = \hat{V}_{conf}(0) + \frac{d\hat{V}_{conf}}{dr}\Big|_{r=0} r + \frac{d^2\hat{V}_{conf}}{dr^2}\Big|_{r=0} \frac{r^2}{2} + \cdots$$

$$\hat{V}_{conf}(r) = \frac{d^2\hat{V}_{conf}}{dr^2}\Big|_{r=0} \frac{r^2}{2} \equiv \frac{\mu\omega^2 r^2}{2} \quad \Rightarrow \quad <\hat{T}> = <\hat{V}_{conf}(r)>$$

$$pR \sim \hbar \;\Rightarrow\; \omega \sim \frac{\hbar}{\mu R^2}$$

Walter Kohn (1923-2016)



Kohn's Theorem 1961

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Cyclotron Resonance and de Haas-van Alphen Oscillations of an Interacting Electron Gas*

WALTER KOHN
University of California at San Diego, La Jolla, California

(Received April 5, 1961)

An electron gas with short-range interactions is considered in the presence of a uniform magnetic field. It is shown that (1) the cyclotron resonance frequency is independent of the interaction; (2) for a two-dimensional gas, the de Haas–van Alphen period is independent of the interaction. The low-lying excited states are briefly discussed.

Kohn's Theorem 1961

Now if the system is placed in a *homogeneous* rotating microwave field,[2] we must add to H the perturbation

$$H' = -\frac{e}{i\omega m}\mathcal{E}_{-}P_{+}e^{-i\omega t}. \tag{12}$$

We see that the perturbation (12) connects the state Ψ_0 with, and only with, the state Ψ_1, so that there results a sharp absorption at the frequency $\omega=\omega_c$.

In summary: Cyclotron resonance is not affected by the interaction U.

Kohn's Theorem for circular parabolic quantum dots

VOLUME 65, NUMBER 1 PHYSICAL REVIEW LETTERS 2 JULY 1990

Quantum Dots in a Magnetic Field: Role of Electron-Electron Interactions

P. A. Maksym[a] and Tapash Chakraborty

Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany

(Received 26 January 1990)

The eigenstates of electrons interacting in quantum dots in a magnetic field are studied. The interaction has important effects on the magnetic-field dependence of the energy spectrum. However, when the confinement potential is quadratic, the optical excitation energies of the many-body system are exactly the same as those of a single electron. This makes the interaction effects difficult to observe directly but they could be seen by measuring the thermodynamic properties of the electrons. This is illustrated with the calculations of the electronic heat capacity.

Kohn’s Theorem for circular parabolic quantum dots

Magnetoabsorption at quantum points

A. O. Govorov and A. V. Chaplik
Institute of Semiconductors Physics, Siberian Branch of the Academy of Sciences of the USSR

(Submitted 7 May 1990)
Pis’ma Zh. Eksp. Teor. Fiz. **52**, No. 1, 681–683 (10 July 1990)

An exact solution is derived for the problem of the frequencies and intensities of the optical absorption lines of a system of magnetized 2D electrons which are confined at a quantum point by a parabolic potential. No restriction is imposed on the interaction between particles.

Kohn's Theorem for asymmetric parabolic quantum dots

RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 42, NUMBER 2 15 JULY 1990-I

Magneto-optics in parabolic quantum dots

F. M. Peeters*

Bellcore, 331 Newman Springs Road, Red Bank, New Jersey 07701-7020

(Received 3 April 1990)

We show that the position of the resonance lines in the magneto-optical absorption spectrum of a quantum dot with a (asymmetric) parabolic confinement potential is independent of the electron-electron interaction and the number of electrons in the quantum dot.

Ellipsoidal quantum dots

J. Phys. D: Appl. Phys. **41** (2008) 162004

IOP FTC

Fast Track Communication

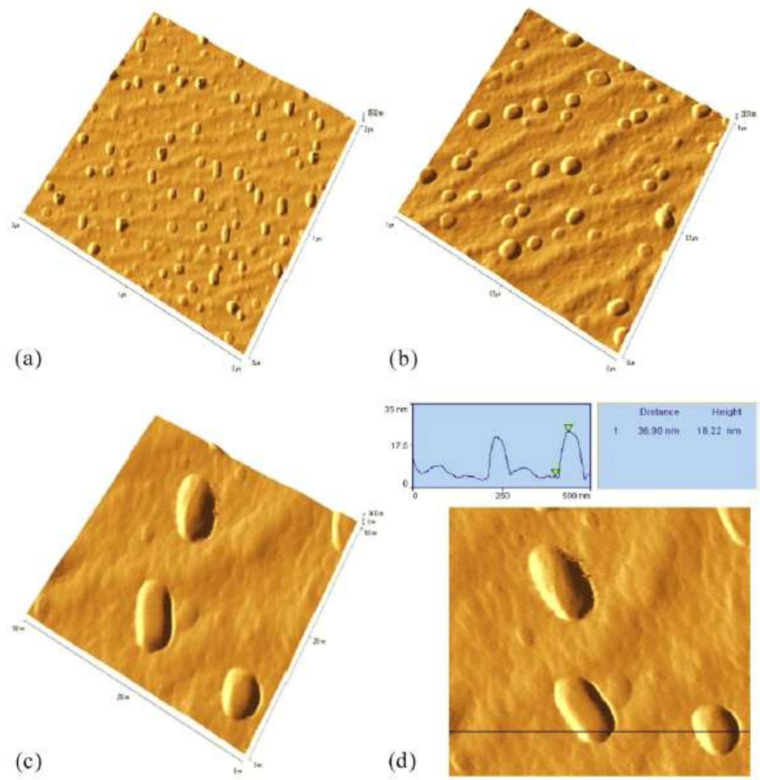
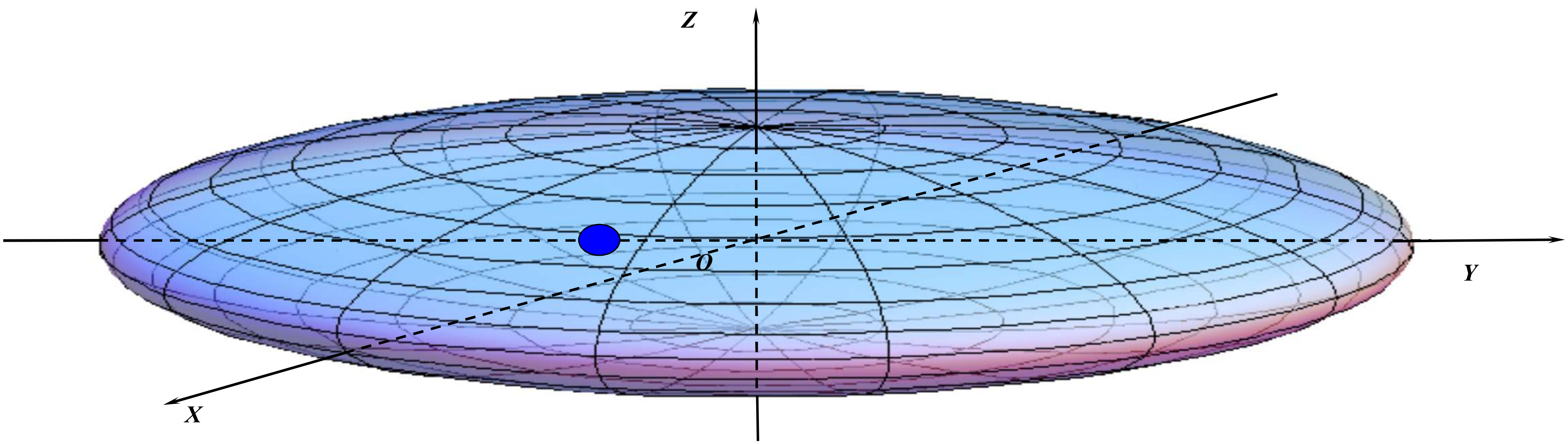


Figure 1. AFM views of LPE grown InAsSbP unencapsulated QDs on InAs(1 0 0) substrate: (*a*) oblique $S = 2 \times 2\,\mu m^2$, (*b*) oblique $S = 1 \times 1\,\mu m^2$, (*c*) oblique $S = 500 \times 500\,nm^2$ and (*d*) plane.

Geometrical adiabatic approximation



Geometrical adiabatic approximation (GAA)

$$\hat{H}'(x,\xi) = \hat{H}_1(x) + \hat{V}(x,\xi)$$

ξ **Coordinate is fixed**

$$\left[\hat{H}_1(x) + V(x,\xi)\right]\Psi_{n_1}(x,\xi) = E_{n_1}(\xi)\Psi_{n_1}(x,\xi)$$

$E_{n_1}(\xi)$ **Eigenvalues**

$\Psi_{n_1}(x,\xi)$ **Eigenstates**

$$\Psi_N = \sum_{n_1} \Psi_{n_1}(x,\xi)\Phi_{Nn_1}(\xi)$$

Full system Wave Function

Essence of GAA method

1st step

$$\left[\hat{H}_1(x)+V(x,\xi)\right]\Psi_{n_1}(x,\xi)=E_{n_1}(\xi)\Psi_{n_1}(x,\xi)$$

$$E_{n_1}(\xi)=U_{eff}(\xi) \qquad \Psi_{n_1}(x,\xi)$$

Essence of GAA method

1st step

$$\left[\hat{H}_1(x)+V(x,\xi)\right]\Psi_{n_1}(x,\xi)=E_{n_1}(\xi)\Psi_{n_1}(x,\xi)$$

2nd step

$$\left[\hat{H}_2(\xi)+U_{eff}(\xi)\right]\Phi_{n_1n_2}(\xi)\approx E_{n_1n_2}\Phi_{n_1n_2}(\xi)$$

$$\Downarrow$$

$$E_{n_1n_2} \qquad \Phi_{n_1n_2}(\xi)$$

Publications

ISSN 1068–3372, Journal of Contemporary Physics (Armenian Academy of Sciences), 2013, Vol. 48, No. 1, pp. 32–36. © Allerton Press, Inc., 2013.
Original Russian Text © D.B. Hayrapetyan, E.M. Kazaryan, H.A. Sarkisyan, 2013, published in Izvestiya NAN Armenii, Fizika, 2013, Vol. 48, No. 1, pp. 48–54.

On the Possibility of Implementation of Kohn's Theorem in the Case of Ellipsoidal Quantum Dots

D. B. Hayrapetyan[a,b], E. M. Kazaryan[a], and H. A. Sarkisyan[a,c]

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

ELSEVIER

Implementation of Kohn's theorem for the ellipsoidal quantum dot in the presence of external magnetic field

CrossMark

D.B. Hayrapetyan[a,b], E.M. Kazaryan[a], H.A. Sarkisyan[a,b,c,*]

Letter from Saint Petersburg (08.12.2016)

Добрый день, Айк!

Привет из снежного Петербурга! Мы сейчас готовим статью, и мне хотелось бы сделать там ссылку на применение параболического потенциала к расчету нижних уровней в квантовых точках. Не мог бы ты прислать какую-нибудь подходящую вашу работу, на которую можно сослаться буквально после слов "...parabolic quantum dot model[]"? Дело в том, что мы, кажется, **обнаружили многочастичные эффекты в спектрах поглощения Ge/Si квантовых точек в дальнем ИК диапазоне. При этом весьма занятный факт состоит в том, что энергия перехода из основного в первые возбужденные состояния многочастичного образования в параболическом потенциале не зависит от количества частиц (!). Что-то подобное у нас получается в спектрах поглощения.**

Спасибо!

Антон Софронов.

Important provisions

1. QD contains few-particle gas (particularly, gas of heavy holes),
2. The effective mass of particles is scalar,
3. The QD has specific geometry, which allows dividing the particle's motion into "fast" and "slow",
4. The incident perturbation on the system is long-wavelength.

All conditions are satisfied!

Publications

nanomaterials

MDPI

Article

Realization of the Kohn's Theorem in Ge/Si Quantum Dots with Hole Gas: Theory and Experiment

Hayk A. Sarkisyan [1,2,3,*], David B. Hayrapetyan [1,2], Lyudvig S. Petrosyan [4], Eduard M. Kazaryan [1,2], Anton N. Sofronov [2], Roman M. Balagula [2], Dmitry A. Firsov [2], Leonid E. Vorobjev [2] and Alexander A. Tonkikh [5]

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Received: 15 November 2018; Accepted: 25 December 2018; Published: 3 January 2019

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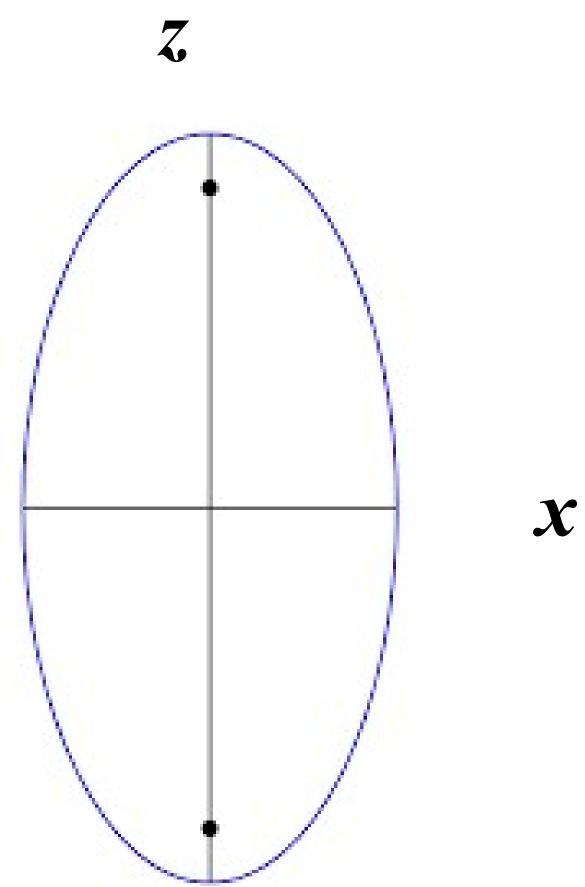
Few-electrons in prolate ellipsoidal quantum dot z

$$V_{\text{int}}(1,\ldots,N) = \sum_{i<j} \upsilon\left(\left|\vec{r}_i - \vec{r}_j\right|\right)$$

$$\hat{V}_{conf}(\vec{r}) = \begin{cases} 0, & \dfrac{x^2+y^2}{a^2} + \dfrac{z^2}{c^2} \le 1 \\ \infty, & \dfrac{x^2+y^2}{a^2} + \dfrac{z^2}{c^2} > 1 \end{cases}, \quad a \ll c$$



One-dimensional pair interacting model

The interparticle interaction in radial direction can be considered weak in comparison with the size quantization taking into account the small thickness of the system in this direction, and therefore, we will consider the interaction between particles in axial direction

$$V_{\text{int}}\left(1,\dots,N\right)=\sum_{i<j}\upsilon\left(\left|z_i-z_j\right|\right)$$

$$\Phi\left(\vec{r}_1,\dots,\vec{r}_N\right)=\phi\left(\vec{\rho}_1\left(z_1\right),\dots,\vec{\rho}_N\left(z_N\right)\right)\cdot\psi\left(z_1,\dots,z_N\right)$$

$$\phi\left(\vec{\rho}_1\left(z_1\right),\vec{\rho}_2\left(z_2\right),\dots,\vec{\rho}_N\left(z_N\right)\right)=\prod_{j=1}^{N}f\left(\vec{\rho}_j\left(z_j\right)\right)$$

Single-particle radial eigenfunction

In radial direction we have single-particle eigenfunction in 2D cyrcular infinitely deep quantum well with diameter:

$$d_\rho = 2a\sqrt{1-\frac{z^2}{c^2}}$$

$$f\left(\vec{\rho}\left(z\right)\right) = C_{n_\rho,|m|} e^{im\varphi} J_m\left(\kappa_{n_\rho,m}\rho\left(z\right)\right), \quad \kappa_{n_\rho,m} = \sqrt{\frac{2\mu E_{n_\rho,m}}{\hbar^2}}$$

n_ρ, m are radial and magnetic quantum numbers

J_m is Bessel functions of the first kind

$C_{n_\rho,|m|}$ is normalization constant

One-dimensional Schrodinger equation

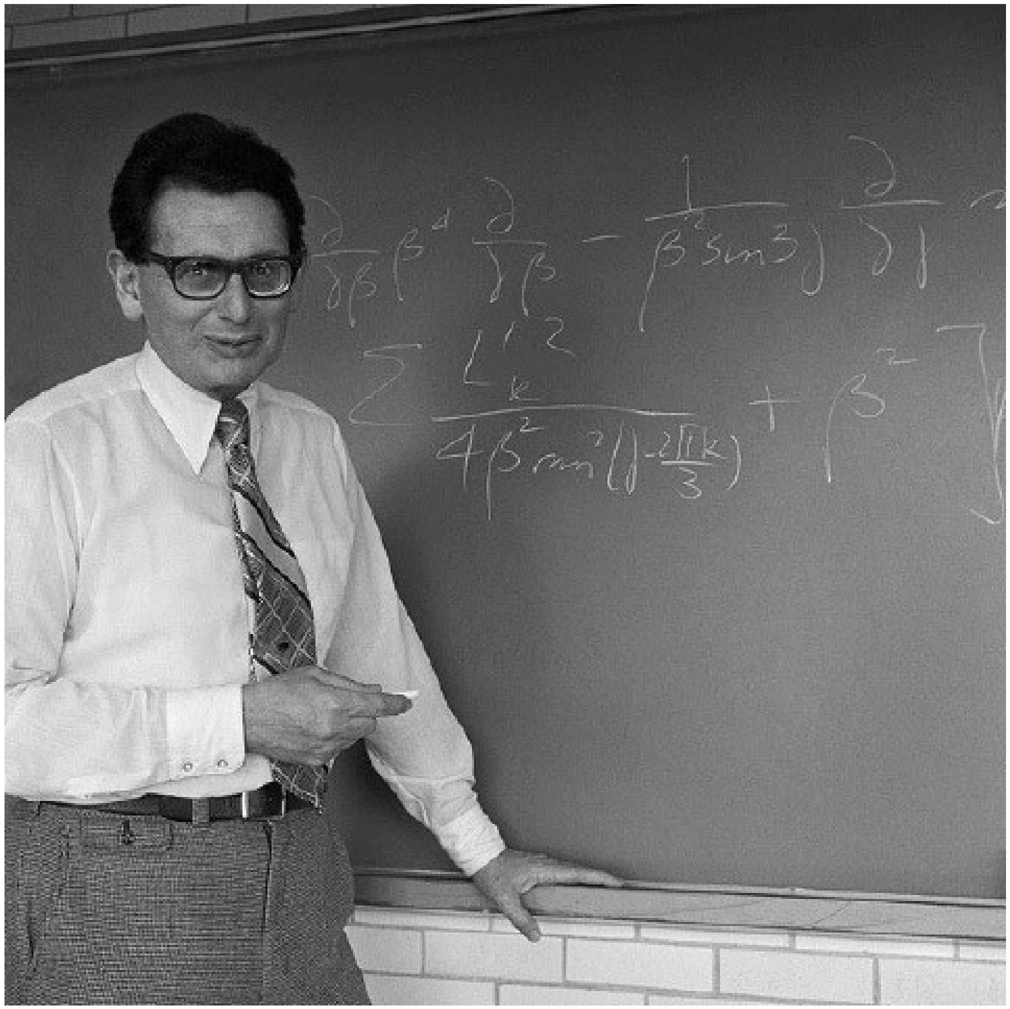
$$V_{conf}\left(z\right)=\frac{\mu\Omega^2 z^2}{2}$$

$\Omega=\frac{1}{\sqrt{2}}\frac{\hbar\alpha_{1,0}}{\mu a c},\quad \alpha_{1,0}$ is the zero of the Bessel function of the first kind

$$-\frac{\hbar^2}{2\mu}\sum_{i=1}^{N}\frac{\partial^2\psi}{\partial z_i^2}+\left\{\frac{\mu\Omega^2}{2}\sum_{i=1}^{N}z_i^2+\sum_{i<j}^{N}\upsilon\left(\left|z_i-z_j\right|\right)\right\}\psi=\left(E-NE_0\right)\psi=\varepsilon\left(N\right)\psi$$

$$\sum_{i<j}^{N}\upsilon\left(\left|z_i-z_j\right|\right)-??!!$$

Marcos Moshinsky (1921-2009)



Moshinsky atom

Am. J. Phys. 36, 52 (1968); doi: 10.1119/1.1974410

How Good is the Hartree-Fock Approximation*

M. Moshinsky
Instituto de Fisica, Universidad de México, México, D. F., México
(Received 12 September 1967)

The problem of two particles in a common harmonic oscillator potential interacting through harmonic oscillator forces is discussed from the standpoint of the Hartree–Fock approximation and compared with the exact solution.

$$H = \tfrac{1}{2}\,[p_1{}^2 + r_1{}^2] + \tfrac{1}{2}\,[p_2{}^2 + r_2{}^2] + \varkappa\,[1/\sqrt{2}\,(\mathbf{r}_1 - \mathbf{r}_2)]^2$$

$$\mathbf{R} = 1/\sqrt{2}\,(\mathbf{r}_1 + \mathbf{r}_2),\ \ \mathbf{r} = 1/\sqrt{2}\,(\mathbf{r}_1 - \mathbf{r}_2)$$

$$H = \tfrac{1}{2}\,[P^2 + R^2] + \tfrac{1}{2}\,[p^2 + (2\varkappa + 1)r^2]$$

Publications

nanomaterials

MDPI

Article

Long-wave Absorption of Few-Hole Gas in Prolate Ellipsoidal Ge/Si Quantum Dot: Implementation of Analytically Solvable Moshinsky Model

David B. Hayrapetyan [1], Eduard M. Kazaryan [1,2], Mher A. Mkrtchyan [1] and Hayk A. Sarkisyan [1,2,*]

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Received: 14 August 2020; Accepted: 15 September 2020; Published: 23 September 2020

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One-dimensional Moshinsky model

$$V_{\text{int}}\left(1,\ldots,N\right)=\gamma\sum_{i<j}^{N}\left(z_i-z_j\right)^2$$

$$-\frac{\hbar^2}{2\mu}\sum_{i=1}^{N}\frac{\partial^2\psi}{\partial z_i^2}+\left\{\frac{\mu\Omega^2}{2}\sum_{i=1}^{N}z_i^2+\gamma\sum_{i<j}^{N}\left(z_i-z_j\right)^2\right\}\psi=\varepsilon\left(N\right)\psi$$

$$\xi=\frac{z}{\sqrt{\frac{\hbar}{\mu\Omega}}}\qquad W\left(N\right)=\frac{E-NE_0}{\hbar\Omega}\qquad g=\frac{\gamma}{\mu\Omega^2}$$

One-dimensional Moshinsky model

Now we transform the original coordinates into Jacobi ones:

$$\begin{cases} Z = \dfrac{\xi_1 + \ldots + \xi_N}{\sqrt{N}} \\ Z_i = \sqrt{\dfrac{i-1}{i}}\left(\xi_i - \dfrac{1}{i-1}\displaystyle\sum_{k=1}^{i-1}\xi_k\right), \qquad i = 2,3,\ldots,N \end{cases}$$

$$\left\{\left(-\frac{1}{2}\frac{\partial^2}{\partial Z^2}+\frac{Z^2}{2}\right)+\sum_{i=2}^{N}\left(-\frac{1}{2}\frac{\partial^2}{\partial Z_i^2}+\frac{1}{2}\tilde{\Omega}^2 Z_i^2\right)\right\}\psi = W(N)\psi$$

$$\tilde{\Omega} = \sqrt{1+2Ng}$$

Energy spectrum and eigenfunction

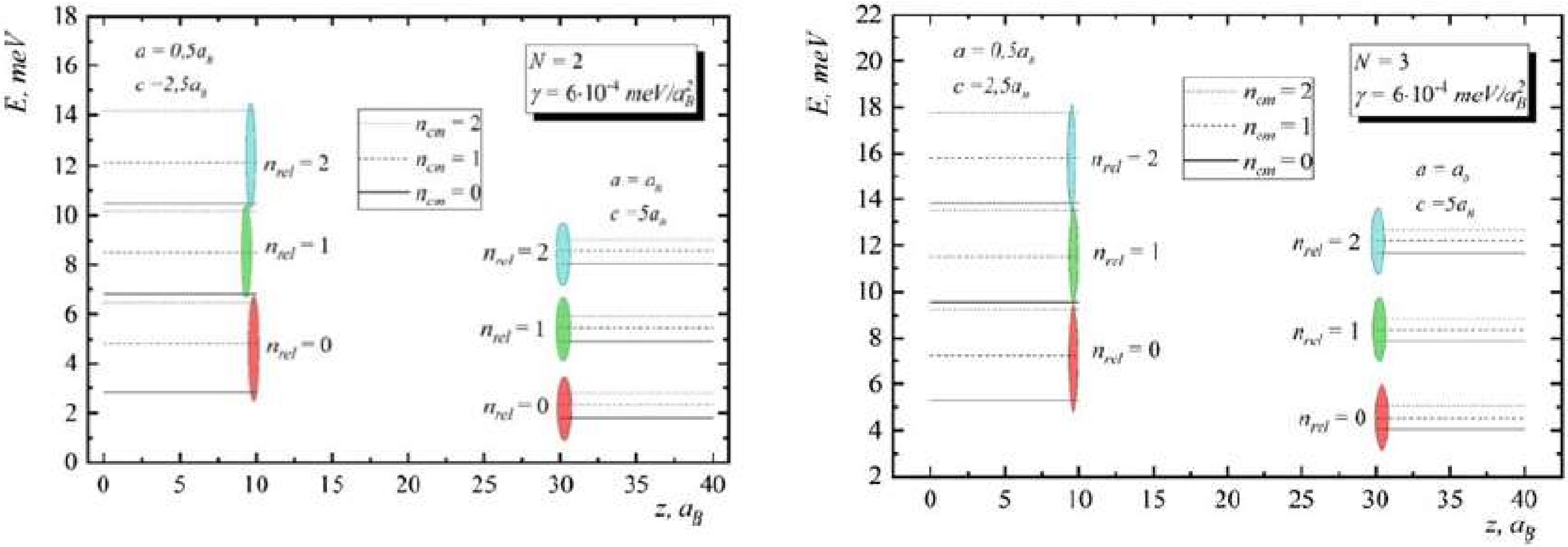
$$E_{n_{cm},\{n_{rel_i}\}} = NE_0 + \hbar\Omega\left(n_{cm} + \frac{1}{2}\right) + \hbar\Omega\tilde{\Omega}\sum_{i=2}^{N}\left(n_{rel_i} + \frac{1}{2}\right)$$

$\{n_{cm}, n_{rel_i}\}$ are quantum numbers for center of mass and relative motion

$$\psi(Z, Z_i) = \frac{1}{\sqrt{2^{n_{cm}} n_{cm}!}}\left(\frac{1}{\pi}\right)^{1/4} e^{-Z^2/2} H_{n_{cm}}(Z) \prod_{i=2}^{N} \frac{1}{\sqrt{2^{n_{rel_i}} n_{rel_i}!}}\left(\frac{\tilde{\Omega}}{\pi}\right)^{1/4} e^{-\frac{\tilde{\Omega}}{2}Z_i^2} H_{n_{rel_i}}\left(\sqrt{\tilde{\Omega}} Z_i\right)$$

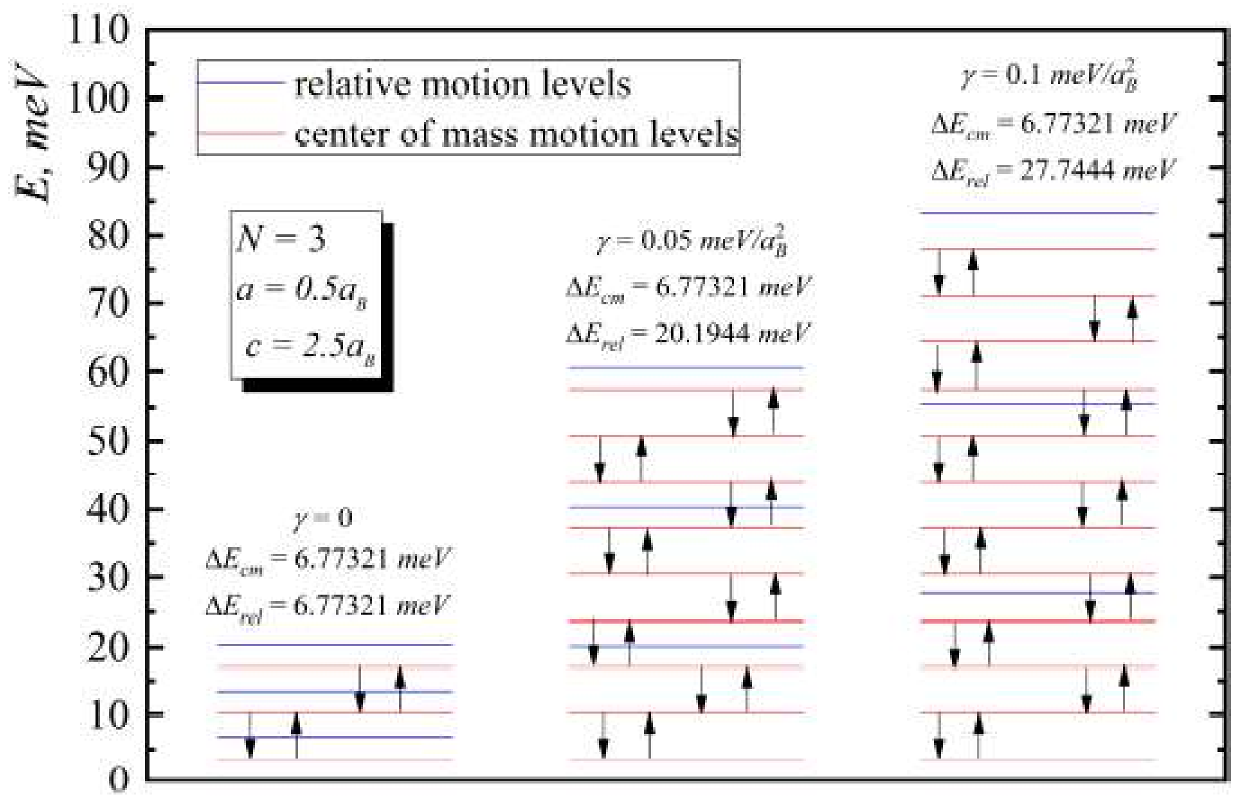
Interaction parameter was calculated by equating the Moshinsky model potential with the Coulomb one and for discussed model.

Energy diagrams for N=2 and N=3



Energy diagram for 2-particle and 3-particle systems

Transitions related to the generalized Kohn theorem



The motion of the center of mass doesn't depend on the number of particles and interaction, and inter-level distance is not being changed !!!!

Two-dimensional quantum lens

Physica E 150 (2023) 115703

Contents lists available at ScienceDirect

Physica E: Low-dimensional Systems and Nanostructures

journal homepage: www.elsevier.com/locate/physe

One- and few-particle optics of the valence band in lens-shaped Ge/Si quantum dots

M.A. Mkrtchyan [a], D.B. Hayrapetyan [a,b], E.M. Kazaryan [a,b], H.A. Sarkisyan [a,b,*], S. Baskoutas [c], D.A. Firsov [b], M. Ya Vinnichenko [b]

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[b] Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia
[c] University of Patras, 265 04, Patras, Greece

ARTICLE INFO

Handling Editor: J. Nitta

Keywords:
Lens-shaped quantum dot
Nonlinear absorption
Second harmonic generation
Hole gas
Moshinsky model
Kohn theorem

ABSTRACT

The theoretical investigation of one- and few-particle states in asymmetric biconvex lens-shaped Ge/Si quantum dot have been considered. Heavy hole transitions in considered structure have been studied. The behaviors of linear and nonlinear absorption coefficients for different quantum dot geometric parameters have been observed. Pair-interacting few-heavy hole gas has been investigated by modeling the interaction potential between holes in the frame of Moshinsky model. The character of long-wave transitions between the center of mass levels of the system have been obtained when Kohn theorem is realized.

Two-dimensional quantum lens

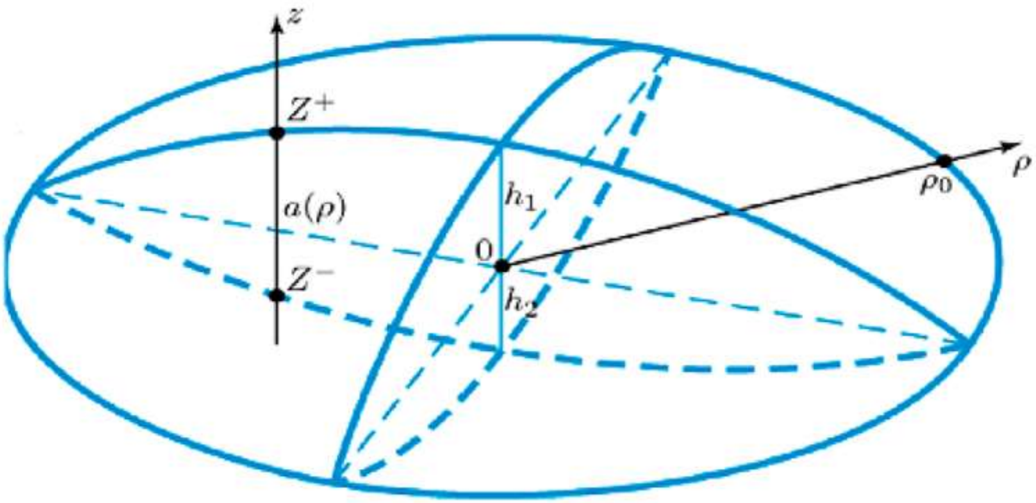


Fig. 1. Asymmetric biconvex lens-shaped QD.

$$U_{conf}(\overrightarrow{r}) = \begin{cases} 0, particle \in QD \\ \infty, particle \notin QD \end{cases}.$$

Two-dimensional Moshinsky model

$$\widehat{H}(1,...,N) = \widehat{H}_x(1,...,N) + \widehat{H}_y(1,...,N),$$

where

$$\begin{cases} \widehat{H}_x(1,...,N) = \dfrac{1}{2m^*}\displaystyle\sum_{j=1}^{N}\widehat{p}_{xj}^2 + \dfrac{m^*\Omega_{n_z=1}^2}{2}\sum_{j=1}^{N}x_j^2 + \gamma\sum_{i<j}^{N}\left(x_i - x_j\right)^2 \\ \widehat{H}_y(1,...,N) = \dfrac{1}{2m^*}\displaystyle\sum_{j=1}^{N}\widehat{p}_{yj}^2 + \dfrac{m^*\Omega_{n_z=1}^2}{2}\sum_{j=1}^{N}y_j^2 + \gamma\sum_{i<j}^{N}\left(y_i - y_j\right)^2 \end{cases}.$$

$$V_{\text{int}}(1,...,N) = \sum_{i<j}^{N}\gamma\left(\left|\overrightarrow{\rho}_i - \overrightarrow{\rho}_j\right|\right)^2 = \gamma\left(\sum_{i<j}^{N}\left(x_i - x_j\right)^2 + \sum_{i<j}^{N}\left(y_i - y_j\right)^2\right)$$

Experiment

ACS APPLIED NANO MATERIALS

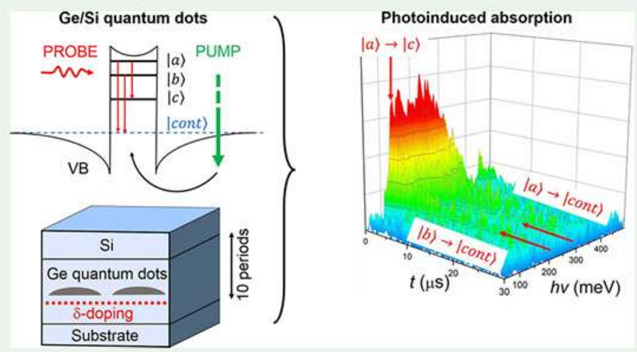
www.acsanm.org | Article

Effect of Doping and Interband Pumping on the Optical Properties of GeSi/Si Quantum Dot Nanostructures for Infrared Detectors

Ratmir V. Ustimenko,* Maxim Ya. Vinnichenko,* Danila A. Karaulov, Hayk A. Sarkisyan, David B. Hayrapetyan, and Dmitry A. Firsov

Cite This: https://doi.org/10.1021/acsanm.4c05251 | Read Online

ABSTRACT: Nanostructures with quantum dots based on GeSi solid solution are promising for the development of optoelectronic devices compatible with modern silicon technology. In this paper, we demonstrate the capabilities of such nanostructures for detecting infrared and terahertz radiation. The working spectral range of nanostructures with GeSi/Si QDs is determined by the energy position of hole levels in the QDs, which is calculated using the quantum box model and confirmed by experimentally measuring the spectra of photoinduced intraband absorption of radiation. Using time-resolved spectroscopy, we found the characteristic times that determine the speed of the detection process associated with the processes of capture and recombination of charge carriers.



KEYWORDS: *quantum dots, GeSi, terahertz, infrared absorption, time-resolved spectra, Kohn theorem*

Experiment

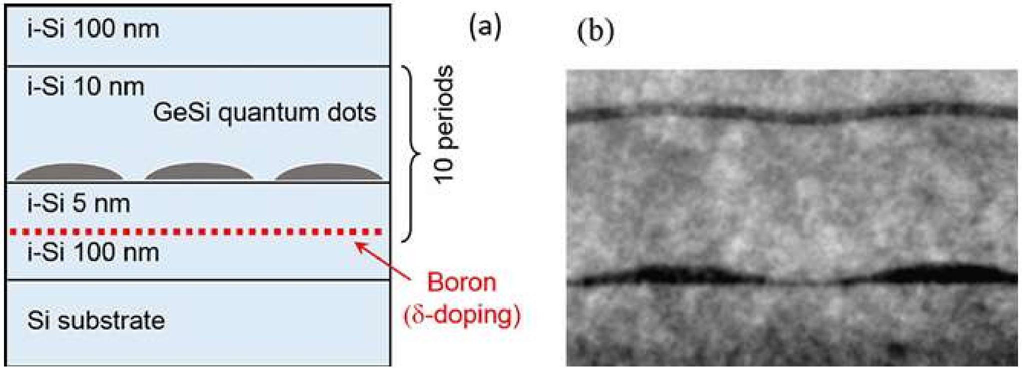


Figure 1. (a) Scheme of experimental samples with GeSi/Si QDs with different concentration of acceptor boron impurity. (b) High resolution TEM image of a cleaved sample of QDs.

Let now a long-wavelength electromagnetic perturbation with the electric component $\vec{\varepsilon}(t) = \vec{\varepsilon}_0 e^{-i\omega t}$ fallsonthesystem. In the mentioned approximation the perturbation operator is written as

$$\hat{H}_1 = -e\vec{\varepsilon}(t)\sum_{i=1}^{N}\vec{\rho}_i = -e\cdot\sqrt{\frac{\hbar}{m^*\Omega_{n_z=1}}}\cdot\sqrt{N}\cdot e^{-i\omega t}\vec{\varepsilon}_0(\zeta_1\cos\vartheta + \zeta_2\sin\vartheta).$$

Experiment

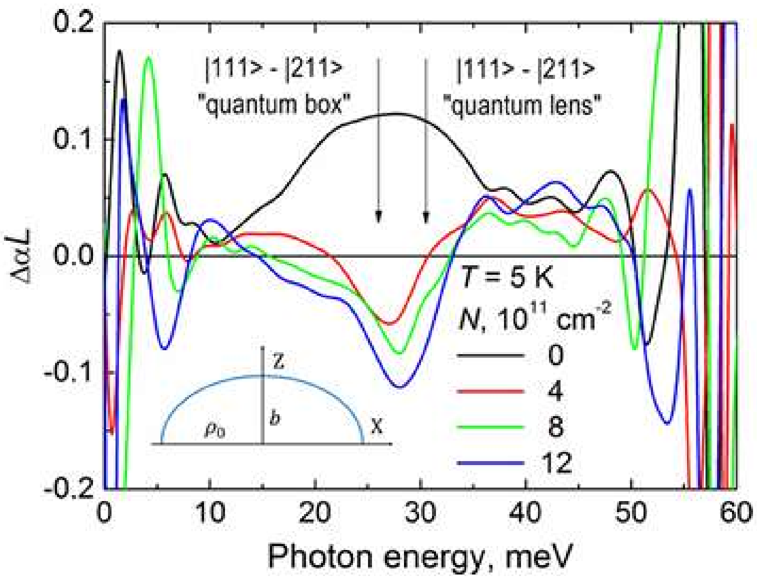


Figure 7. Photoinduced absorption spectra of THz radiation of QDs with different doping levels. The inset schematically shows the QD profile in the form of a flattened half-ellipse.

$$\hbar\omega_{\text{theory}} = \frac{\pi\hbar^2}{m_h(b^3R)^{0.5}} \tag{9}$$

where R is the radius of curvature of the QD surface expressed through the length of the base of the QD $2\rho_0$ and through its height b:

$$R = \frac{b^2 + \rho_0^2}{2b} \tag{10}$$

For large QDs with a height b = 3.0 nm and a base $2\rho_0$ = 18 nm, which are filled first, $\hbar\omega_{\text{theory}}$ = 30.5 meV. In Figure 6, this position is shown by the arrow $|111\rangle \rightarrow |211\rangle$ "quantum lens".

Johnson-Payne model

VOLUME 67, NUMBER 9 PHYSICAL REVIEW LETTERS 26 AUGUST 1991

Exactly Solvable Model of Interacting Particles in a Quantum Dot

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(Received 1 April 1991)

A simple, yet physically reasonable, model is presented which describes an arbitrary number of interacting particles in a quantum dot. Exact analytic expressions are obtained for the energy spectrum as a function of particle number and magnetic field.

PACS numbers: 73.20.Dx, 73.20.Mf, 73.40.Kp

Recent progress in semiconductor technology now allows the fabrication of zero-dimensional structures, called quantum dots [1-8], where islands of two-dimensional electrons are laterally confined by an imposed potential. The number of electrons in a quantum dot can be varied over a considerable range, and recent far-infrared absorption [1-3], capacitance spectroscopy [5], and conductance [8] measurements have yielded data with rich structure. However, a full understanding of the experimental results requires knowledge of the many-electron energy spectrum for a quantum dot. The few existing theoretical treatments of the energy spectrum have been computationally intensive and have employed either the Hartree approximation [9], whose neglect of exchange and correlation effects may be significant in quantum dots [10], or direct numerical diagonalization [11,12], which is

space would, of course, vary as $|\mathbf{r}_i-\mathbf{r}_j|^{-1}$. However, in quantum-dot structures, the form of $V(\mathbf{r}_i,\mathbf{r}_j)$ is modified in a nontrivial way by the presence of image charges in adjacent layers of gates. Furthermore, the wave functions of the electrons confined in the quantum dots have a small but finite extent perpendicular to the x-y plane [19]. This results in a slight smearing of the electron charges along the perpendicular (z) direction. Hence the interparticle repulsion will tend to saturate at small interparticle separations. We choose as our model for the interaction

$$V(\mathbf{r}_i,\mathbf{r}_j)=2V_0-\tfrac{1}{2}m^*\Omega^2|\mathbf{r}_i-\mathbf{r}_j|^2, \tag{1}$$

where m^* is the particle effective mass; V_0 and Ω are positive parameters within the model. Reasonable values for V_0 and Ω will be discussed later in the Letter.

Creation and annihilation operators

The operators, introduced above, allows to separate the motion of the centre mass from the relative motion!!

$$\hat{H}^{2D} = \hat{H}^{2D}_{cm} + \hat{H}^{2D}_{rel}$$

$$\hat{H}^{2D}_{cm} = \hbar\Omega\left[\hat{A}^{+}\hat{A}^{-} + \frac{1}{2}\right] + \hbar\Omega\left[\hat{B}^{+}\hat{B}^{-} + \frac{1}{2}\right]$$

$$\hat{H}^{2D}_{rel} = \frac{\hbar\Omega_0}{N}\left[\sum_{i<j}\hat{a}^{+}_{ij}\hat{a}^{-}_{ij} + \sum_{i<j}\hat{b}^{+}_{ij}\hat{b}^{-}_{ij} + N(N-1)\right] + N(N-1)V_0$$

For both Hamiltonians, the ground states (vacuum) are determined according to equation:

$$\hat{D}\left|0\right\rangle \equiv 0 \qquad \hat{D} = \left\{\hat{A}^{-}, \hat{B}^{-}, \hat{a}^{-}_{ij}, \hat{b}^{-}_{ij}\right\}$$

Energy spectrum and wave function

$$\left|0\right\rangle \sim \exp\left(-\frac{N\mu\Omega}{2\hbar}\left(X^2+Y^2\right)\right)\exp\left(-\frac{\mu\Omega_0}{2N\hbar}\sum_{i<j}\left(x_{ij}^2+y_{ij}^2\right)\right)$$

$$E_0=\hbar\Omega+\hbar\Omega_0\left(N-1\right)+N\left(N-1\right)V_0\left(\textit{vacuum state}\right)$$

$$\left|N_A,N_B,\alpha_{ij},\beta_{ij}\right\rangle=\left(\hat{A}^+\right)^{N_A}\left(\hat{B}^+\right)^{N_B}\prod_{i<j}\left(\hat{a}_{ij}^+\right)^{\alpha_{ij}}\left(\hat{b}_{ij}^+\right)^{\beta_{ij}}\left|0\right\rangle \quad \left(\left\{N_A,N_B,\alpha_{ij},\beta_{ij}\right\}\textit{ are integers}\right)$$

$$E_{\alpha_{ij},\beta_{ij}}^{N_A,N_B}=E_0+\hbar\Omega\left(N_A+N_B\right)+\hbar\Omega_0\left\{\sum_{i<j}\left(\alpha_{ij}+\beta_{ij}\right)\right\}$$

Publications

Heliyon 10 (2024) e34762

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Research article

Two-dimensional pair-interacting hole gas thermodynamics: Exactly solvable Moshinsky model for lens-shaped quantum dots

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Nuclear Instruments and Methods in Physics Research A 1073 (2025) 170251

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Few-particle gas in strongly oblate asymmetric ellipsoidal quantum dot

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Thank you